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A non-iterative design procedure for supplemental brace-damper systems in single–degree–of–freedom systems

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SUMMARY

In this paper a method for designing supplemental brace-damper systems in single–degree–of–freedom (SDOF) structures is presented. We include the affects of the supporting brace stiffness in the dynamic response by using a viscoelastic Maxwell model. Based on the study of a SDOF under ground excitation, we propose a non-iterative design procedure for simultaneously specifying both the damper and brace while assuring a desired structural performance. It is shown that to increase the damper size beyond the value delivered by the proposed criteria will not provide any improvement but actually worsen the structural response. The design method presented here shows excellent agreement with the FEMA 273 design approach but offers solutions closer to optimality.

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KEY WORDS: Damper sizing, Maxwell model, supplemental damping, bracedamper systems

1. INTRODUCTION

For reducing unwanted vibrations in civil engineering structures, passive systems are valued for their inherent stability and reliability [1, 2]. Most of the solutions proposed for sizing dampers have been based on iterative approaches [3–6]. In a practical context however, procedures offering a direct solution have the advantage of being quick and easy to apply and in some situations could be more convenient to use than those based on numerical optimisation. The lack of simple design guidelines has motivated many researchers to investigate practical procedures intended to guide the engineer in selecting commercially available dampers. Some examples include the simplified design methodology discussed in [7] and [8]. Those procedures are intended to give the preliminary damper's sizing based on the structural properties and seismic hazard. Comparable design procedures have also been proposed in the literature (see e.g. [9–11]).

In this paper, we present a method for designing a supplemental brace–damper assembly in a SDOF system. Unlike more commonly used approaches, we include the effects of the supporting brace stiffness in the dynamic response. This is an important effect, because as will be shown, when the brace stiffness is included, an optimal damper value can be defined beyond which it is counter–productive to increase the damper size. Based on the study of a SDOF system under ground seismic excitation, we propose a non–iterative procedure with two easy–to–use criteria for the preliminary design of brace–damper systems. This delivers not only the damper size but also the brace stiffness that is needed to ensure a desired structural behaviour in terms of the overall damping ratio.

The brace compliance reduces the displacement effectively transferred to the damper and introduces a shift in phase and frequency. These issues have been studied by several researchers

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seeking more accurate design procedures. Some works consider the brace stiffness by way of a loss factor that reduces the velocity transfer to the damper (see e.g. [12]); or consider a viscoelastic behaviour for the damper and formulate gradient–based optimisation algorithms as in [13]. The previous work most directly related to this paper was published in 2011 by Chen and Chai [14]. They proposed an iterative design method based on the minimisation of some performance indices. Our work differs from theirs in that we present a procedure in a few consecutive steps that does not require any iteration or time–consuming optimisation process. Another related work was presented in 1998 by Fu and Kasai [15]. Their formulation assumes steady–state harmonic motion of an SDOF oscillator fitted with a brace–damper arrangement at its undamped natural frequency. Instead, our approach is not restricted to steady–state motion, being still valid for any kind of dynamic excitation.

The proposed method may also be applicable to multi–degree–of–freedom (MDOF) structures that can be represented by an equivalent SDOF system. Note that under certain circumstances harmonic motion at the natural frequency or a displacement shape could be assumed to transform a MODF structure to a SDOF system with good approximation of its dynamic behaviour.

2. BRACED SINGLE DEGREE OF FREEDOM STRUCTURE

Figure 1. a) Structure with an added brace–damper system. b) Maxwell model.

We assume the case of a linear fluid viscous damper in serial arrangement with its supporting brace, as shown in Figure 1. This configuration can be represented by the Maxwell model and described by the first–order differential equation in formula (1b) where f_d is the force in the arrangement, c_d is the coefficient of the viscous damper, k_b is the horizontal stiffness of the brace, x is the total deformation of the arrangement and $(\dot{\ })$ denotes the derivative with respect to time t.

The system dynamics under earthquake excitation can be described by means of the system of equation (1), where m, c, k represent the mass, damping and stiffness of the structure; x is the structural displacement and $\ddot{x}_g$ is the ground acceleration.

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) + f_d(t) = -m\ddot{x}_g(t) \tag{1a}$$

$$f_d(t) + \frac{c_d}{k_b}\dot{f}_d(t) = c_d\dot{x}(t) \tag{1b}$$

We calculate the system transfer function for the structural displacement with respect to the ground acceleration. Two dimensionless parameters, α and β, are used to represent the brace stiffness and the damper size as ratios of the structural stiffness and damping ratio respectively.

$$\omega_n^2 = \frac{k}{m}; \qquad \xi = \frac{c}{2m\omega_n}; \qquad \alpha = \frac{k_b}{k}; \qquad \beta = \xi\frac{c_d}{c} = \frac{c_d}{2\sqrt{km}} \tag{2}$$

By substituting the above parameters into equation (1), applying the Laplace transform method and solving the system for the transfer function $T(s) = \frac{X(s)}{\ddot{X}_g(s)}$, we obtain the following expression where s is the Laplace variable.

$$T(s) = \frac{-(2\beta s + \alpha\omega_n)}{2\beta s^3 + \omega_n(\alpha + 4\xi\beta)s^2 + 2\omega_n^2(\xi\alpha + \beta + \alpha\beta)s + \alpha\omega_n^3} \triangleq \frac{N(s)}{D(s)} \tag{3}$$

We use the poles of (3) for estimating the overall damping ratio of the system. The poles are the values of s that cause the transfer function to become infinite. The third–order polynomial $D(s)$ can be factorised in terms of its roots to get:

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$$D(s) = \left(\frac{s}{\omega_n} - w\right)\left(\frac{s}{\omega_n} - (u + vj)\right)\left(\frac{s}{\omega_n} - (u - vj)\right) = 0 \tag{4}$$

where u, v and w are functions of the system parameters α, β and ξ. The overall structural damping ratio, ξ_T, can be evaluated by using the conjugate poles:

$$\xi_T = -u\left(u^2 + v^2\right)^{-\frac{1}{2}} \tag{5}$$

which is independent of ω_n, see (4). Moreover, note that u and v are solely functions of ξ, α and β, so that, for a particular value of the structural damping ratio ξ, we can obtain the total damping of the system as a function of α and β.

2.1. *Added damping map*

It is not possible to derive an explicit solution for ξ_T in terms of α and β. However it is possible to obtain a damping map as a function of α and β for a constant value of ξ. To show this, let us consider the example of a SDOF system with structural damping ratio of 5%.

By solving the third–order polynomial $D(s) = 0$ and applying equation (5), we can draw the surface and contour plot shown in Figure 2. This plot represents the additional damping ratio ξ_D, defined as $\xi_D = \xi_T - \xi$. Now, as β is a function of ξ the surface will change for different ξ values. Therefore we have introduced an generalised parameter β^*, (which for $\xi = 5\%$ has the relationship $\beta^* = \beta$) so that this one plot can be used for any ξ value.

Our design method is based on the following observations from Figure 2:

- To achieve a certain value of ξ_D there exists a minimum value of α, that is a minimum brace stiffness, that needs to be provided.
- The near vertical zone on the contour curve can be used to define the point at which, for a certain ξ_D, a reduction in the support stiffness would require an increased damper size.
- For any constant value of α there exists a unique value of β^* that maximises the additional damping ratio. Note that increasing β^* beyond that limit will worsen the structural response.

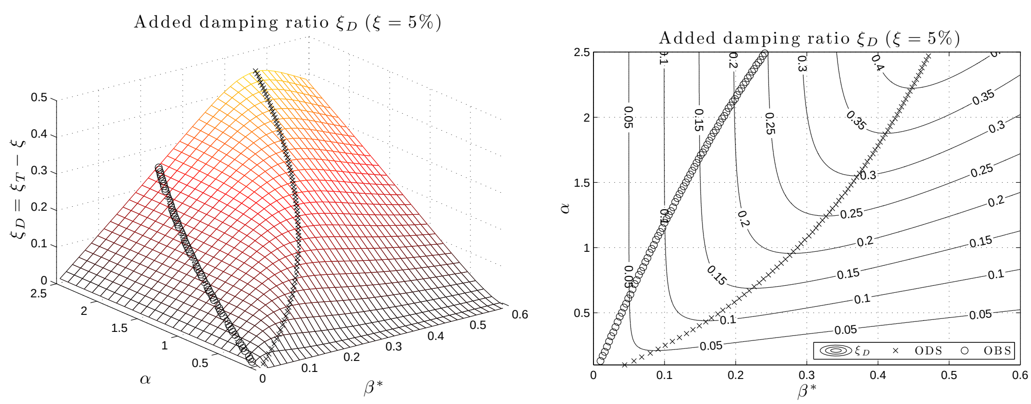

Figure 2. Added damping ratio map for a structure with $\xi = 5\%$.

2.2. *Criteria for sizing the brace and damper*

We now propose a procedure for designing the supplemental brace-damper system. It can be seen from Figure 2 that for a fixed α value the optimal (i.e. highest ξ_D) damping value occurs at the minima of the contour curve. In Figure 2 such points are represented by black crosses and correspond to a fitted curve in the (α, β^*, ξ_D) space defined by the crossing of the surfaces given by:

$$\begin{aligned} \alpha &= 3.9\xi_D^2 + 4.0\xi_D \\ \beta^* &= 2.81\xi_D - 0.314\alpha + 0.017 \end{aligned} \tag{6}$$

We call this the *Optimum Damper Size (ODS) criterion.*

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Alternatively, if it is required to keep the damper sizes as small as possible, but still achieve a certain ξ_D value we can fit a curve to give the minimum α for each contour. Therefore we select α such that the change of ξ_D with respect to α is less than 0.5%: $(\{\min(\alpha)|\partial_\alpha \xi_D < 0.5\%\})$. For a given added damping this results in the minimum damper size required. Such points can be defined as (see black circles in Figure 2):

$$\begin{aligned} \beta^* &= -0.17\xi_D^2 + 1.02\xi_D \\ \alpha &= -53.0\xi_D + 64.4\beta^* + 0.03 \end{aligned} \tag{7}$$

This is called the *Optimum Brace Stiffness (OBS) criterion*.

Equations (6) and (7) have been derived from the case of $\xi = 5\%$. For arbitrary values of structural damping, the damping maps can be scaled with respect of β without significant loss of accuracy by:

$$\beta^* = (0.4\xi + 0.98)\beta \tag{8}$$

In summary the four-step design procedure for a brace-damper system is:

1. Choose the required damping ratio of the SDOF system after adding the damper.
2. Determine the values of the pair (α, β^*) for the desired added damping by using either equation (6) or (7) in accordance with the criterion selected.
3. Make correction to the parameter β^* by applying formula (8) and get β.
4. Use the definitions in (2) to obtain the damper size c_d and the brace stiffness k_b.

This procedure can be used to calculate either (i) the optimal damper required for a fixed brace stiffness, or (ii) the minimum brace stiffness required by the structure to achieve a desired overall damping ratio. We also note that the load carrying capacity of the brace still needs to be able to withstand the maximum force generated by the damper. In addition, the size of an equivalent nonlinear damper capable of dissipating the same amount of energy as the linear device delivered here can be estimated by considering the energy-equivalence principle presented in [1].

2.3. *Numerical example.*

Consider a one–floor structure with mass of 1000Kg, lateral stiffness of 150kN/m and damping equal to 3% of critical. Suppose that we want to add a brace–damper arrangement to increase the damping up to 25%. Following the steps described above for the ODS criterion we have that:

The desired additional damping ratio ξ_D is 22%. From equation (6) the values for $\alpha = 1.0688$ and then $\beta^* = 0.2996$ can be calculated straightaway. Then, from equation (8), β can be estimated to be: 0.3020; and finally, by using the definitions in (2) we can obtain the required size for both the brace and damper as: $k_b = 160.32$ kN/m; and $c_d = 7.40$kNs/m. By repeating the procedure considering the OBS criterion, equation (7) can be used to obtain the reference values $\beta^* = 0.2162$ and $\alpha = 2.2915$. As before, we use equation (8) to account for the structural damping ratio and use the definitions in (2) to get: $k_b = 343.73$kN/m and $c_d = 5.34$kNs/m.

Figure 3a shows the Bode diagram of the transfer function in (3) for this example. Note that because of the additional damping provided, the peak response of the original system is strongly reduced when considering either design criteria. Whenever these values are exceeded no significant improvement is obtained in the structural response. Figure 3b shows the dynamic response when doubling either the damper size or brace stiffness estimated by the ODS and the OBS criterion.

The values delivered from the proposed procedure are compared against the provisions defined in FEMA 273. The equations 9–30 in these guidelines provide the approximate design formula [16]:

$$\xi_d = \frac{T \sum_j c_{dj} \cos^2\theta_j \phi_{rj}^2}{4\pi \sum_i (\frac{w_i}{g}) \phi_i^2} \tag{9}$$

where T is the period, θ_j is the angle of inclination of the device, ϕ_{rj} is the relative displacement of the device, w_i is the reactive weight of floor and ϕ_i is the floor displacement. After substituting the known values of the example, the equation (9) can be solved to get: $c_d = 5.39$kNs/m. Note that one could assume steady–state harmonic motion and consider the transformation from the Kelvin model to the Maxwell model as suggested in [17]. We supposed a small stiffness in the Kelvin model (5kN/m) so that the damper size of the equivalent Maxwell model lay within 0.5% of the difference

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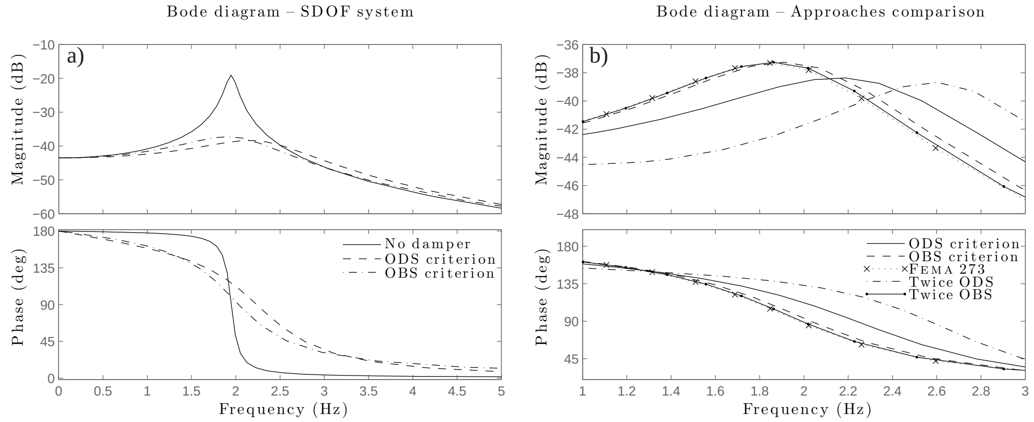

Figure 3. (a) Dynamic response of the SDOF structure with and without the brace–damper system and (b) comparison among different design approaches.

from the damper size of the Kelvin model. This was used to estimate the near–rigid brace stiffness (e.g., 876.5kN/m) and damper size (e.g., 5.42kNs/m) that match the pure dashpot behaviour assumed in FEMA 273. Note however that the results are highly dependent on the dummy stiffness initially assumed and that they are strictly valid at the fundamental frequency used in the transformation.

The design outputs are evaluated by simulating the dynamic response of the structure under the earthquake records NGA–1048 and NGA–173 taken from the PEER ground motion database [18]. We use the set of performance indices presented in Table I to measure the level of reduction achieved for each design criteria for the two earthquakes. Figure 4 shows the responses of the structure when excited by the Northridge earthquake. We also include in Figure 4b the response of the structure without dampers but with an inherent structural damping of 25% to show that the design target was achieved. The plots show that both design criteria ODS and OBS offer solutions that behave satisfactorily and are comparable to the other commonly accepted design approach. Note that the ODS criterion delivers a value of k_b that is about the structural stiffness k and OBS around twice k,

Method	k_b (kN/m)	c_d (kNs/m)	$\max_{\forall t,i}\left(\frac{\|\ddot{x}_i(t)\|_c}{\|\ddot{x}_i^{\max}\|_u}\right)$		$\max_{\forall t}\left(\frac{\lvert x_{\text{top}}(t)\rvert_c}{\lvert x_{\text{top}}^{\max}\rvert_u}\right)$		$\max_{\forall t,i}\left(\frac{\lvert f_{d_i}(t)\rvert}{100kN}\right)$	
			EQ1	EQ2	EQ1	EQ2	EQ1	EQ2
ODS	160.32	7.40	0.345	0.521	0.406	0.613	1.617	0.785
OBS	343.73	5.34	0.217	0.403	0.409	0.594	1.341	0.735
FEMA	876.56	5.42	0.201	0.341	0.396	0.590	1.354	0.746

c: structure with added brace–damper; u: uncontrolled structure

Table I. Design outputs and performance indices for each sizing criteria. EQ1: Northridge (NGA–1048). EQ2: Imperial Valley (NGA–173)

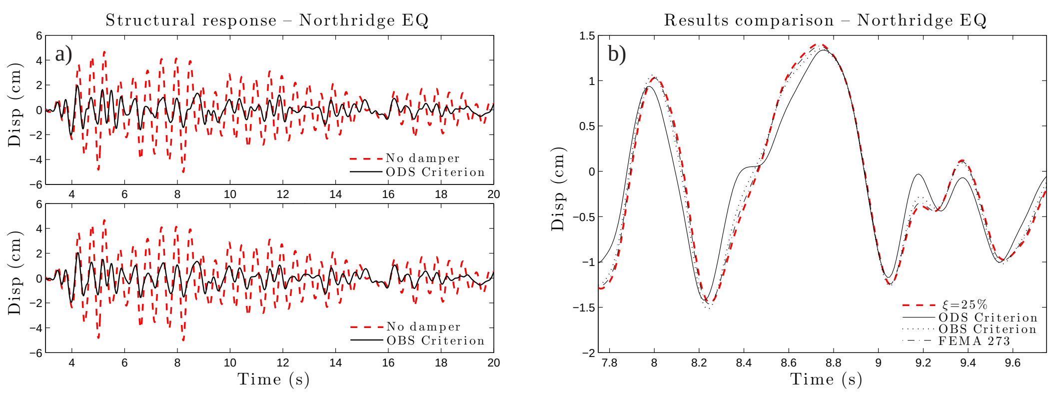

Figure 4. Floor response of the structure with brace–damper system. (a) ODS and OBS criteria; (b) Comparison against structure with no damper but inherent structural damping ratio of 25%.

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while the transformation of the FEMA provision results in a k_b of almost 6 times k. Furthermore, notice how OBS almost perfectly tracks the dynamic behaviour of the system with damping ratio of 25%. This shows that the proposed design methodology can deliver solutions close to optimality.

CONCLUSIONS

In this paper we have proposed a simplified non–iterative procedure in four steps that allows the engineer to simultaneously obtain both the size of linear viscous fluid damper and its supporting brace stiffness when considering SDOF structures. We defined two different criteria named the Optimum Damper Size and the Optimum Brace Stiffness (depending on whether the minimum size for the brace or damper is wanted) to satisfy the desired structural performance expressed in terms of the overall damping ratio. It is shown that to increase the damper size or brace stiffness beyond the value delivered by either criteria will not provide any significant improvement in the structural response. Instead, increasing the damper size beyond the value delivered by ODS will worsen the structural response. The results are also compared with a commonly accepted design approach showing excellent agreement in terms of performance but offering solutions closer to optimality.

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